Physics 4A
Chapters 16 and 17: Traveling Waves and Superposition

"People of mediocre ability sometimes achieve outstanding success because they don't know when to quit. Most people succeed because they are determined to." – George Allen

"*You're on the road to success when you realize that failure is only a detour.****"*** – unknown

"One important key to success is self-confidence. An important key to self-confidence is preparation." – Arthur Ashe

Reading: sections 16.1 – 16.5, 16.7, 16.9; 17.1 – 17.3, 17.5 – 17.8

Outline:

⇒ introduction to waves
- mechanical, electromagnetic, and matter waves
- wave speed

⇒ traveling waves
- transverse and longitudinal waves
- amplitude and phase
- wavelength and frequency
- wave speed
- sinusoidal waves

⇒ waves speed on a stretched string
⇒ waves in two- and three-dimensions

⇒ the principle of superposition
⇒ standing waves
- Standing waves on a string

⇒ interference
- interference in one-dimension
- constructive and destructive interference
- phase difference
- interference in two- and three-dimensions
- beats

Problem Solving

Many of the problems involving sinusoidal waves on a string deal with the relationships $v = \lambda f = \lambda / T = \omega / k$, where v is the wave speed, λ is the wavelength, f is the frequency, T is the period, ω is the angular frequency, and k is the angular wave number. Typical problems might give you the wavelength and frequency, then ask for the wave speed, or might give you the wave speed and period, then ask for the wavelength or angular wave number.

Sometimes the quantities are given by describing the motion. For example, a problem might tell you that the string at one point takes a certain time to go from its equilibrium position to maximum displacement. This, of course, is one-fourth the period. In other problems the frequency of the source (a person's hand or a mechanical oscillator, for example) might be given. You must then recognize that the frequency of the wave is the same as the frequency of the source.

For some problems you must be able to write an expression for the displacement as a function of position and time for a sinusoidal wave. Since it has the form $D(x,t) = A\sin(kx \pm \omega t + \phi_0)$, you must be able to determine k, ω, and ϕ_0 from data given in the problem statement. The sign in front of ωt is determined by the direction of travel. In other problems you may be given the displacement as a function of coordinate and time and asked to identify various quantities.

Some problems deal with the wave speed. For waves on a string, the fundamental equation is $v = \sqrt{T_s / \mu}$, where T_s is the tension in the string and μ is the linear density of the string.

Some problems dealing with standing waves on a string give you the amplitude, frequency, and angular wave number (or related quantities) for the traveling waves and ask for the standing wave pattern. Instead of the wavelength or angular wave number of the traveling waves you might be told the distance between successive nodes or successive antinodes. Double it to find the wavelength. If you are told the distance between a node and a neighboring antinode, multiply it by 4 to find the wavelength.

If a standing wave is generated in a string with both ends fixed, the wave pattern must have a node at each end of the string. This means the length L of the string and the wavelength l of the traveling waves must be related by $L = n\lambda/2$, where n is an integer. If one end is fixed and the other is free, the fixed end is a node and the free end is an antinode, so the length must be an odd multiple of $\lambda/4$

For fully destructive interference the two waves are out of phase by an odd multiple of π rad; for fully constructive interference their phases differ by zero or a multiple of 2π rad. In some cases the phase difference comes about because the two waves travel different distances from their sources to the detector. This phase difference is given by $\phi = 2\pi \Delta d / \lambda$, where Δd is the difference in the distances traveled and λ is the wavelength. There may be an additional phase difference because the sources are not in phase with each other. You may be given the difference in the distances traveled and asked for the phase difference or the amplitude of the resultant wave at the detector.

Some problems deal with the production of beats by two sinusoidal sound waves with nearly the same frequency. You may be given the frequency f_1 of one of the waves and the beat frequency f_{beat}, then asked for the frequency f_2 of the other wave. Since $f_{beat} = |f_1 - f_2|$, it is given by $f_2 = f_1 \pm f_{beat}$. You require more information to determine which sign to use in this equation.

One way to give this information is to tell you what happens to the beat frequency if f_1 is increased (or decreased). If the beat frequency increases when f_1 increases, then f_1 must be greater than f_2 and $f_2 = f_1 - f_{beat}$. If the beat frequency decreases, then f_1 must be less than f_2 and $f_2 = f_1 + f_{beat}$.

Nearly all Doppler shift problems can be solved using

$$f' = \left(\frac{v \pm v_D}{v \pm v_S} \right) f$$

where v is speed of sound, v_S is the speed of the source, v_D is the speed of the observer, f is the frequency of the source, and f' is the frequency detected by the observer. The upper sign in the numerator refers to a situation in which the observer is moving toward the source; the lower sign refers to a situation in which the observer is moving away from the source. The upper sign in the denominator refers to a situation in which the source is moving toward the observer; the lower sign refers to a situation in which the source is moving away from the observer. Remember that all speeds are measured relative to the medium in which the sound is propagating. You might be given the velocities and one of the frequencies, then asked for the other frequency. In other situations you might be given the two frequencies and one of the velocities, then asked for the other velocity. In all cases, simple algebraic manipulation of the equation will produce the desired expression.

Mathematical Skills

Partial derivatives.

Partial derivatives are important for understanding much of this chapter. The displacement $y(x,t)$ of a string carrying a wave is a function of two variables, the coordinate x of a point on the string and the time t. You may differentiate with respect to either variable.

The notation $\partial y(x,t)/\partial x$ stands for the derivative of y with respect to x, with t treated as a constant. Similarly, $\partial y(x,t)/\partial t$ means the derivative of y with respect to t, with x treated as a constant. The result of either differentiation may again be a function of x and t. For example, $\partial \sin(kx - \omega t) / \partial x = k\cos(kx - \omega t)$ and $\partial \sin(kx - \omega t) / \partial t = -\omega \cos(kx - \omega t)$.

You should understand the physical significance of a partial derivative as well as be able to evaluate it, given the function. The partial derivative $\partial y(x,t)/\partial x$ gives the slope of the string at the point x and time t; here you are evaluating the rate at which the displacement changes with *distance* along the string at some instant of time. For this to be meaningful the definition must make use of displacements for slightly separated points on the string *at the same time*, in the limit as the separation tends to zero. That is why t is treated as a constant.

The partial derivative $\partial y(x,t)/\partial t$ gives the string velocity at the point x and time t; here you are evaluating the rate at which the displacement changes with *time* at a given point. Clearly this is associated with the displacement *of the same point* but at slightly different times, in the limit as the time interval approaches zero. That is why x is treated as a constant.

GENERAL PRINCIPLES

The Wave Model

This model is based on the idea of a traveling wave, which is an organized disturbance traveling at a well-defined **wave speed** v.

- In transverse waves the displacement is perpendicular to the direction in which the wave travels.

- In longitudinal waves the particles of the medium are displaced parallel to the direction in which the wave travels.

A wave transfers **energy,** but no material or substance is transferred outward from the source.

Two basic classes of waves:

- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through a vacuum.

For mechanical waves, such as sound waves and waves on strings, the speed of the wave is a property of the medium. Speed does not depend on the size or shape of the wave.

IMPORTANT CONCEPTS

The **displacement** D of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave's displacement as a function of position at a single instant of time.

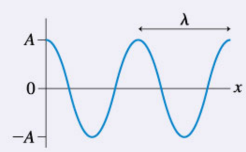
- A history graph shows the wave's displacement as a function of time at a single point in space.

For a transverse wave on a string, the snapshot graph is a picture of the wave. The displacement of a longitudinal wave is parallel to the motion; thus the snapshot graph of a longitudinal sound wave is *not* a picture of the wave.

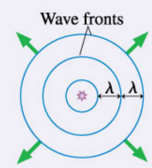
Sinusoidal waves are periodic in both time (period T) and space (wavelength λ):

$$D(x, t) = A\sin[2\pi(x/\lambda - t/T) + \phi_0]$$
$$= A\sin(kx - \omega t + \phi_0)$$

where A is the **amplitude,** $k = 2\pi/\lambda$ is the **wave number,** $\omega = 2\pi f = 2\pi/T$ is the **angular frequency,** and ϕ_0 is the **phase constant** that describes initial conditions.



One-dimensional waves



Two- and three-dimensional waves

The fundamental relationship for any sinusoidal wave is $v = \lambda f$.

APPLICATIONS

- **String** (transverse): $v = \sqrt{T_s/\mu}$
- **Sound** (longitudinal): $v = \sqrt{B/\rho} = 343$ m/s in 20°C air
- **Light** (transverse): $v = c/n$, where $c = 3.00 \times 10^8$ m/s is the speed of light in a vacuum and n is the material's **index of refraction**

The wave intensity is the power-to-area ratio: $I = P/a$

For a circular or spherical wave: $I = P_{source}/4\pi r^2$

The sound intensity level is

$$\beta = (10\text{ dB})\log_{10}(I/1.0 \times 10^{-12}\text{ W/m}^2)$$

The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency f_0 emitted.

Approaching source

$$f_+ = \frac{f_0}{1 - v_s/v}$$

Observer approaching a source

$$f_+ = (1 + v_o/v)f_0$$

Receding source

$$f_- = \frac{f_0}{1 + v_s/v}$$

Observer receding from a source

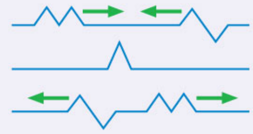
$$f_- = (1 - v_o/v)f_0$$

The Doppler effect for light uses a result derived from the theory of relativity.

GENERAL PRINCIPLES

Principle of Superposition

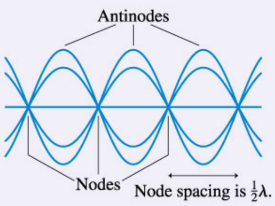
The displacement of a medium when more than one wave is present is the sum at each point of the displacements due to each individual wave.



IMPORTANT CONCEPTS

Standing Waves

Standing waves are due to the superposition of two traveling waves moving in opposite directions.

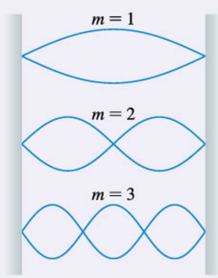


The amplitude at position x is

$$A(x) = 2a \sin kx$$

where a is the amplitude of each wave.

The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.

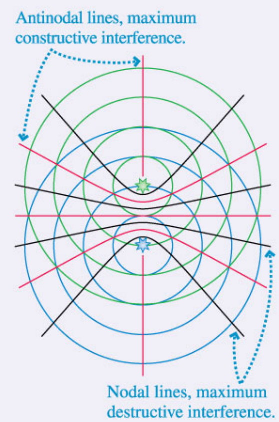


Standing waves on a string

Solving Interference Problems

Maximum constructive interference occurs where crests are aligned with crests and troughs with troughs. The waves are in phase.

Maximum destructive interference occurs where crests are aligned with troughs. The waves are out of phase.



MODEL Model the wave as linear, circular, or spherical.

VISUALIZE Find distances to the sources.

SOLVE Interference depends on the **phase difference** $\Delta\phi$ between the waves:

$$\text{Constructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$$

$$\text{Destructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \left(m + \tfrac{1}{2}\right) \cdot 2\pi$$

Δr is the path-length difference of the two waves, and $\Delta\phi_0$ is any phase difference between the sources. For identical (in-phase) sources:

$$\text{Constructive: } \Delta r = m\lambda \qquad \text{Destructive: } \Delta r = \left(m + \tfrac{1}{2}\right)\lambda$$

ASSESS Is the result reasonable?

APPLICATIONS

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends:

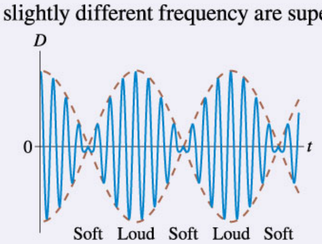
$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = mf_1 \qquad m = 1, 2, 3, \ldots.$$

The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = mf_1 \qquad m = 1, 3, 5, 7, \ldots.$$

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.

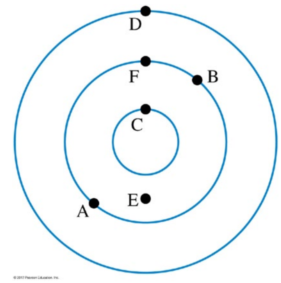


The beat frequency between waves of frequencies f_1 and f_2 is

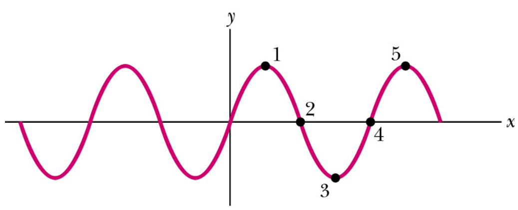
$$f_{\text{beat}} = |f_1 - f_2|$$

Questions and Example Problems from Chapter 16 and 17

Conceptual Question 16.9
The figure shows the wave fronts of a circular wave. What is the phase difference between **(a)** points A and B, **(b)** points C and D, and **(c)** points E and F?



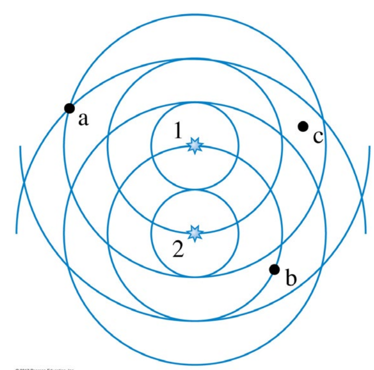
Conceptual Question 16.A
In the figure below, five points are indicated on a snapshot of a sinusoidal wave. What is the phase difference between point 1 and **(a)** point 2, **(b)** point 3, **(c)** point 4, and **(d)** point 5? Answer in radians and in terms of the wavelength of the wave. The snapshot shows a point of zero displacement at x = 0. In terms of the period T of the wave, when will **(e)** a peak and **(f)** the next point of zero displacement reach x = 0?



Conceptual Question 16.B
The following four waves are sent along strings with the same linear densities (*x* is in meters and *t* is in seconds). Rank the waves according to **(a)** their wave speed and **(b)** the tensions in the strings along which they travel, greatest first:

(1) y_1 = (3 mm) sin(x – 3t)
(2) y_2 = (6 mm) sin(2x – t)
(3) y_3 = (1 mm) sin(4x –t)
(4) y_4 = (2 mm) sin(x – 2t)

Conceptual Question 17.9
The figure shows the circular waves emitted by two in-phase sources. Are a, b, and c points of maximum constructive interference, maximum destructive interference, or in between?



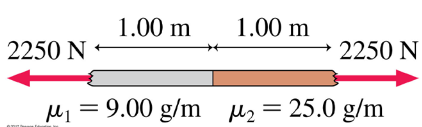
Problem 16.2
A 25 g string is under 20 N of tension. A pulse travels the length of the string in 50 ms. How long is the string?

Problem 16.12
The displacement of a wave traveling in the positive x-direction is $D(x,t) = (3.5\,cm)\sin(2.7x - 124t)$, where x is in m and t is in s. What are the **(a)** frequency, **(b)** wavelength, and **(c)** speed of this wave?

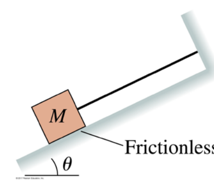
Problem 16.41
A friend of yours is loudly singing a single note at 400 Hz while racing toward you at 25.0 m/s on a day when the speed of sound is 340 m/s. **(a)** What frequency do you hear? **(b)** What frequency does your friend hear if you suddenly start singing at 400 Hz?

Problem 16.44
A mother hawk screeches as she dives at you. You recall from biology that female hawks screech at 800 Hz, but you hear the screech at 900 Hz. How fast is the hawk approaching?

Problem 16.59
A wire is made by welding together two metals having different densities. The figure shows a 2.00-m-long section of wire centered on the junction, but the wire extends much farther in both directions. The wire is placed under 2250 N tension, then a 1500 Hz wave with an amplitude of 3.00 mm is sent down the wire. How many wavelengths (complete cycles) of the wave are in this 2.00-m-long section of the wire?



Problem 16.60
The string in the figure has linear density μ. Find an expression in terms of M, μ, and θ for the speed of waves on the string.



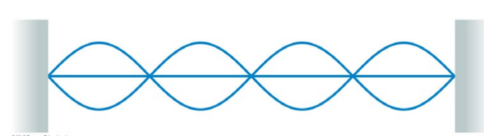
Problem 16.A
The equation of a transverse wave traveling along a very long string is $y = 6.0\,\sin(0.020\pi x + 4.0\pi t)$, where x and y are expressed in centimeters and t is in seconds. Determine **(a)** the amplitude, **(b)** the wavelength, **(c)** the frequency, **(d)** the speed, **(e)** the direction of propagation of the wave, and **(f)** the maximum transverse speed of a particle in the string. **(g)** What is the transverse displacement at x = 3.5 cm when t = 0.26 s?

Problem 16.B
The equation of a transverse wave on a string is

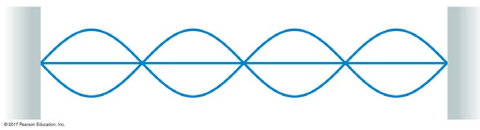
$$y = (2.0\text{ mm})\sin[(20\text{ m}^{-1})x - (600\text{ s}^{-1})t]$$

The tension in the string is 15 N. **(a)** What is the wave speed? **(b)** Find the linear density of the string in grams per meter.

Problem 17.6
The figure shows the standing wave on a 2.0-m-long string that has been fixed at both ends and tightened until the wave speed is 40 m/s. What is the frequency?



Problem 17.7
The figure shows a standing wave on a string that is oscillating at 100 Hz. **(a)** How many antinodes will there be if the frequency is increase to 200 Hz? **(b)** If the tension is increased by a factor of 4, at what frequency will the string continue to oscillate as a standing wave that looks like the one in the figure?



Problem 17.22
Two loud speakers emit sound waves along the x-axis. The sound has maximum intensity when the speakers are 20 cm apart. The sound intensity decreases as the distance between the speakers is increased, reaching zero at a separation of 60 cm. **(a)** What is the wavelength of the sound? **(b)** If the distance between the speakers continues to increase, at what separation will the sound intensity again be a maximum?

Problem 17.41
A violinist places her finger so that the vibrating section of a 1.0 g/m string has a length of 30 cm, then she draws her bow across it. A listener nearby in a 20° C room hears a note with a wavelength of 40 cm. What is the tension in the string?

Problem 17.44
A 75 g bungee cord has an equilibrium length of 1.20 m. The cord is stretched to a length of 1.80 m, then vibrated at 20 Hz. This produces a standing wave with two antinodes. What is the spring constant of the bungee cord?

Problem 17.62
Two loudspeakers emit sound waves along the x-axis. A listener in front of both speakers hears a maximum sound intensity when speaker 2 is at the origin and speaker 1 is at x = 0.50 m. If speaker 1 is slowly moved forward, the sound intensity decreases and then increases, reaching a maximum when speaker 1 is at x = 0.90 m. **(a)** What is the frequency of the sound? Assume v_{sound} = 340 m/s. **(b)** What is the phase difference between the speakers?

Problem 17.A
In the figure below, a French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic. The French sub moves at speed v_F = 50.00 km/h, and the U.S. sub at v_{US} = 70.00 km/h. The French sub sends out a sonar signal (sound wave in water) at 1.00×10^3 Hz. Sonar waves travel at 5470 km/h. (a) What is the signal's frequency as detected by the U.S. sub? (b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?